

# Bi-parametric traveltimes and stacking apertures for reflection and diffraction enhancement

J. H. Faccipieri\* (CEPETRO/UNICAMP), T. A. Coimbra (CEPETRO/UNICAMP), L. -J. Gelius (UiO) and M. Tygel (IMECC/UNICAMP)

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#### Abstract

The quality of stacking results (e.g., event enhancement, continuity and noise reduction) is strongly influenced not only by the chosen traveltime, but also by the employed stacking apertures. In this work, we consider two, socalled, diffraction-stack traveltimes, together with corresponding stacking apertures, designed to enhance reflections or diffractions. The first traveltime under consideration is the zero-offset (ZO), common-reflectionsurface (CRS) diffraction traveltime, that is obtained from the general ZO CRS traveltime in the case that the target reflector reduces to a point, referred as the single-squareroot (SSR) traveltime. The second is the double-squareroot (DSR) traveltime, well established in time migration. The SSR and DSR traveltimes will be given specific stacking apertures based on the Projected Fresnel Zone (PFZ). SSR with small apertures in midpoint, produced comparable results with reduced computational cost as compared with the ones of conventional CRS with fullparameter reflection traveltimes. In both situations, reflections are enhanced and diffractions attenuated. DSR with large midpoint apertures yields stacked sections in diffractions are enhanced and reflections which attenuated. The aperture size for optimal stacking is quantified by means of the PFZ that corresponds to the events (reflections or diffractions) under consideration. Synthetic and field data confirm the good potential of the proposed approach for image-quality improvement.

## Introduction

Stacking is probably the tool of most widespread use in seismic processing. It takes advantage of the large redundancy of seismic dataset to "clean" it. This process, significantly enhance the signal-to-noise ratio, as well as having events (say, reflections/diffractions) better suited for more reliable interpretation. Stacking traveltimes are designed to enhance desirable events by constructive interference, while attenuating undesired events or noise by destructive interference.

Diffraction traveltimes occupy a prominent place as stacking operators because of their robustness and simplicity, their best example being its role in Kirchhofftype migration. A good reason for the great success of diffraction-stack traveltime stems from Huygen's principle, in which the reflection response of a reflector can be thought as a superposition of the responses of point scatterers densely distributed on the reflector.

A number of factors influences good stacking. These include (a) a stacking operator (traveltime) that is able to accurate follow (approximate) desired events, (b) a coherence measure that is able to quantify how well the stacking operator approximates the desired events, (c) possible weights to improve the stacking and/or producing more meaningful amplitudes and (d) carefully chosen apertures that are able to focus stacking where only constructive interference takes place.

Here, we examine the influence of midpoint aperture under the use of two diffraction operators, namely (a) the ZO CRS diffraction traveltime, referred throughout as the single-square-root (SSR) traveltime and (b) the doublesquare-root (DSR) diffraction traveltime. SSR is a simplified version of the conventional ZO CRS traveltime, which results when the target reflector reduces to a (diffraction) point. DSR, of widespread use in time migration, is an exact expression of a point-diffraction response in a homogeneous medium. In fact, one can show that SSR is the second-order, Taylor-polynomial representation of DSR.

In this work, we find that small apertures in midpoint produce enhanced reflections and attenuated diffractions. On the other hand, large apertures in midpoint produce enhanced diffractions and attenuated reflections. In the case of reflection enhancement, stacking results obtained using SSR are comparable to the ones obtained (at a higher computational cost) by conventional, full-parameter CRS. However, use of the DSR traveltime with large midpoint apertures, leads to significant enhancement of diffractions together with attenuation of reflections.

The aperture size for optimal stacking in both situations is quantified by means of the projected Fresnel Zone (PFZ) that corresponds to the events (reflections or diffractions) under consideration. Synthetic and field data confirm a good imaging potential of the proposed approach.

# CRS, SSR and DSR traveltimes

In the following, we introduce and briefly discuss the CRS, SSR and DSR traveltimes. Our task is greatly facilitated by their widespread and routine use in seismic processing.

## **CRS traveltime**

The CRS method (Jäger et al., 2001) is based on the generalized hyperbolic traveltime, which uses first and second derivatives with respect to midpoint and half-offset in the vicinity of a selected central or reference ray.

In the general case of a finite-offset central ray, the number of parameters of the CRS traveltime is five and fourteen for the 2D and 3D situations, respectively. Here we adopt the simpler case (of much more widespread use) in which the central ray is a ZO ray and also assuming non-converted data and isotropic media. In this situation, the number of CRS parameters reduces to, respectively, three and eight parameters for 2D and 3D datasets. The 2D ZO CRS hyperbolic traveltime reads

$$t(m,h) = \sqrt{[t_0 + A\Delta m]^2 + B\Delta m^2 + Ch^2}$$
(1)

where (m, h) denotes the midpoint and half-offset coordinates of a source receiver pair in the vicinity of the ZO central ray of coordinates  $(m_0, h = 0)$ . Coefficients (CRS parameters) *A*, *B* and *C* are given by

$$A = \frac{\partial t}{\partial m}, \quad B = t_0 \frac{\partial^2 t}{\partial m^2} \quad \text{and} \quad C = t_0 \frac{\partial^2 t}{\partial h^2},$$
 (2)

all derivatives being evaluated at  $m = m_0$  and h = 0. As can be readily verified, the hyperbolic CRS traveltime of Equation (1) is directly obtained from its parabolic counterpart (namely a second-order Taylor polynomial of traveltime, instead of traveltime squared), by squaring both sides and retaining the terms up to second order only.

It is also well known that the coefficient, B, in Equation (1), is by far the most unstable parameter, being attached to the so-called normal (N) wave and indirectly related to the curvature of the reflector at the normal-incident-point (NIP). This heavily contrasts with the good behavior exhibited by the remaining parameters, A and C, interpreted as slowness of the central ZO ray at its emergence point and the normal moveout (NMO) velocity, respectively. As the CRS parameter B has the most unstable estimation, it would be attractive if, at least for initial estimations and lateral velocity variations, one could use a traveltime not dependent on that parameter.

# SSR traveltime

In order to avoid complications involved to the estimation of all CRS parameters, we propose to use a simplified version of Equation (1), in which we set B = C. The resulting expression, referred simply as single-squareroot (SSR) traveltime, is given by

$$E_{SSR}(m,h) = \sqrt{[t_0 + A\Delta m]^2 + C[\Delta m^2 + h^2]}$$
. (3)

As well known (e.g. Duveneck, 2004), the above traveltime readily follows from the full CRS counterpart in the case the target reflector reduces to a (diffraction) point. Substitution of full CRS traveltime with the SSR traveltime for stacking is not a new strategy (see, e.g., Garabito et al., 2001). In fact, the SSR traveltime has a much longer tradition as a form of diffraction stack used in Kirchhoff migration. As shown below, in spite of the fact that the traveltime Equation (3) is naturally attached to diffractions, we find that such traveltime can be very well suited to reflections, as long as proper apertures, in both midpoint and offset, are chosen. More specifically, for reflection enhancement, a small aperture in midpoint should be used combined with a large aperture in offset.

#### DSR traveltime

Because of its Taylor expansion character, SSR fails to approximate the diffraction events when large apertures in midpoint and offset are considered. Trying to avoid such limitation, one can use smaller apertures in offset, which diminishes the benefits of redundancy. Moreover, in many cases, required near offsets are even not available in the dataset. As a remedy to overcome such drawbacks, we use a different diffraction traveltime equation, also defined in terms of CRS parameters, namely the double-square-root (DSR). That is given by

$$t_{DSR}(m,h) =$$

$$\frac{1}{2} \left[ \sqrt{[t_0 + A\Delta s]^2 + C\Delta s^2} + \sqrt{[t_0 + A\Delta g]^2 + C\Delta g^2} \right]$$
(4)

where  $\Delta s = m - h - m_0$  and  $\Delta g = m + h - m_0$ . The rule of thumb behind the use of DSR is that, as opposed to SSR, it provides an exact point-diffraction traveltime in homogeneous media. As a consequence, at least for mild-to-moderate laterally velocity variations, DSR should be expected to well approximate diffractions in apertures comparable to the Fresnel zone associated with the measurement configuration.

#### Stacking apertures

In the following, we examine the problem of choosing apertures optimally designed for enhancing reflection or diffraction events. Our analysis uses the concept and properties of the so-called Projected Fresnel Zone (PFZ). Introduced in Schleicher et al. (1997), the PFZ is given by

$$\left|t_{Ref} - t_{Dif}\right| \le \frac{w}{2},\tag{5}$$

where  $t_{Ref}$  and  $t_{Dif}$  represents the reflection and diffraction traveltimes, respectively. Finally, *w* is the pulse length.

One key observation is that, by its very definition, the size of the PFZ is small for reflections and large for diffractions in midpoint direction. As shown in Faccipieri et al. (2013) and Asgedom et al. (2013), the use of large (midpoint) apertures in SSR can be very effective for imaging diffraction energy. However, the definition of what can be considered small and large should be better defined in order to achieve optimal results for diffraction or reflection enhancement. It should be noted that Asgedom et al. (2013), in the framework of common-offset (CO) CRS, introduced a traveltime curve tailored for diffractions. That traveltime generalizes DSR in a horizontally layered replacement medium. In this situation, proper bounds on the CRS parameters can be easily obtained, with the aperture size being directly related to the displacement of the central point to the horizontal coordinate of the diffraction point.

Our aim now is to express the PFZ Inequality (5) in terms of the coefficients (CRS parameters) of the CRS, SSR and DSR traveltimes. For that purpose, it is convenient to introduce the parabolic version of these traveltimes, meaning the second-order Taylor polynomial approximations of the square-root Equations (1), (3) and (4), respectively. Denoted by  $t_{CRS}^{par}$ ,  $t_{SSR}^{par}$ , and  $t_{DSR}^{par}$ , such parabolic traveltimes are given by

$$t_{CRS}^{par} = t_0 + A\Delta m + \frac{1}{2t_0} [B\Delta m + Ch], \qquad (6)$$

and

t

$$\sum_{SSR}^{par} = t_{DSR}^{par} = t_0 + A\Delta m + \frac{1}{2t_0}C[\Delta m + h].$$
(7)

Expressing the traveltimes  $t_{Ref}$  and  $t_{Dif}$  by their parabolic approximations (6) and (7) in the PFZ Inequality (5) yields

$$|\Delta m| \le \sqrt{\frac{wt_0}{|B-C|}}.$$
(8)

The inequality (8) relates the midpoint aperture with the CRS parameters B and . As seen below, such inequality will be taken as a starting point to define the midpoint apertures to be used to stack reflections or diffractions.

We note that the above relations pose no restrictions on offset. Our proposal here is to consider, for offset apertures, the same ones that are used in conventional common-midpoint (CMP) method, those being limited only by second-order approximation, stretch factors and critical angles.

## Apertures for reflections

In principle, the right-hand-side of inequality (8) provides a natural candidate for midpoint aperture designed to enhance reflections. However, if SSR (which depends on *A* and *C* only) is the traveltime of choice, the dependence of parameter *B* is to be eliminated. For that aim, we introduce the following heuristic assumptions: 1)  $|B| \le |C|$ on a reflection event and 2) An initial approximation of parameter *C* is known. In this case, the minimum stacking aperture,  $\delta_{Ref}^{(m)}$ , can be considered as the case where B = -C, which leads to

$$\delta_{Ref}^{(m)} = |\Delta m| \le \sqrt{\frac{wt_0}{2C}} \,. \tag{9}$$

Recalling that

$$C = \frac{4}{v_{NMO}^2},\tag{10}$$

where  $v_{NMO}$  denotes the normal-moveout (NMO) velocity. Substituting Equation (10) into Equation (9) we obtain

$$\delta_{Ref}^{(m)} \le \frac{v_{NMO}}{2} \sqrt{\frac{wt_0}{2}}.$$
 (11)

We observe that, once the stacking apertures in midpoint,  $\delta_{Ref}^{(m)}$  and in half-offset,  $\delta_{Ref}^{(h)}$ , are defined, the estimation of A and C can be performed using SSR traveltime of Equation (3). The stacking is then performed with the same apertures used in the estimation of parameters. The difference between the aperture sizes of the original PFZ and  $\delta_{Ref}^{(m)}$  is illustrated in Figure (1). Figure (2) shows a schematic view of the stacking apertures proposed for reflection enhancement.

#### Apertures for diffractions

We now address the problem of finding a counterpart aperture that is adequate for diffraction enhancement. In the same way as in our previous discussion on reflection enhancement, we base our discussion on the Fresnel inequality (5) which has led to inequality (8). In the case of diffraction, we have that, strictly speaking,  $t_{Ref} = t_{Dif}$ , so that, consequently, the midpoint aperture is infinite. Under this circumstance, our strategy is to take the largest possible midpoint aperture, as long as the stacking traveltime (e.g., SSR or DSR) remains a reliable approximation of the one of the event under consideration (see Figure 3). The above argument justifies the choice of the DSR traveltime, as it provides a better approximation of diffraction traveltimes for large midpoints and also offsets.

To select optimal midpoint and half-offset apertures,  $\delta_{Dif}^{(m)}$  and  $\delta_{Dif}^{(h)}$ , to enhance diffractions using the DSR traveltime, we adopt the following (also heuristic) strategy: For each sample,  $(m_0, t_0)$  on which the stacking will performed, we take,

$$\delta_{Dif}^{(h)} = \delta_{CMP}^{(h)}.$$
 (12)

In other words, the aperture employed to estimate the parameter *C* using the CMP configuration. For the midpoint aperture,  $\delta_{Dif}^{(m)}$ , we propose the choice

$$\delta_{Dif}^{(m)} = \delta_{Dif}^{(h)} = \delta_{CMP}^{(h)}.$$
(13)

This means that we take equal apertures in midpoint and half-offset. Figure (4) shows a schematic view of the stacking apertures proposed for diffraction enhancement.

## Results

The proposed approach was applied to a 2D marine real dataset acquired offshore in Brazil with 4ms of time sampling, 12.5 m between Common Midpoint (CMP) gathers with maximum fold of 60 traces. The preprocessing steps applied on this data set can found in Faccipieri et al. (2013).

In order to demonstrate the effectiveness of the SSR estimation and stacking to enhance reflections, we processed the data by (a) conventional CRS, based on the CRS traveltime of three parameters, *A*, *B* and *C* of Equation (1) and (b) the alternative approach, which employs the SSR of two parameters, *A* and *C*, as given by Equation (3). In both situations, we assume that velocity analysis has been previously carried out. In this way, an estimation of NMO-velocities,  $v_{NMO}$ , and offset (CMP) apertures,  $\delta_{Ref}^{(h)}$  are supposedly available. We also assume that the dominant frequency of the data, *w*, has been already estimated.

Under these circumstances, we perform, for both CRS and SSR situations, global estimation of parameters. The last step for our imaging is stacking and here the choice of aperture is crucial. In both cases, we used the apertures in midpoint and half-offset as prescribed by Equation (11).

Figure (5) compares three illustrative stacked traces under the use of CRS and SSR, respectively. The corresponding entire sections are displayed in Figure (6). The results are very similar, with the SSR stack slightly better, showing less high-frequency noise. However, since the proposed minimum aperture reduces the influence of parameter B one can state that if a larger



Figure 1: PFZ for a ZO reflection event with pulse length w at a given point  $(m_0, t_0)$ , highlighted in green. Note how the midpoint aperture for the SSR moveout,  $\delta_{Ref}^{(m)}$ , well adjusts small region of the reflection event.



Figure 2: Schematic representation of the proposed apertures in midpoint  $(\delta_{Ref}^{(m)})$  and half-offset  $(\delta_{Ref}^{(h)})$  direction for reflection enhancement.

aperture in midpoints were used, the resulting stacked section should be better. Figure 7 shows a stacked section obtained with the CRS traveltime with the double of the minimum aperture used on the previous example. Note that the resolution were compromised and the reflectors heavily smoothed.

The results for diffraction enhancement using the proposed aperture in midpoint are shown on Figure 8 for SSR and DSR traveltimes. The apertures in offset direction for the SSR traveltime were three times smaller them the ones used with the DSR traveltime to ensure both traveltimes yield reliable approximations. Nevertheless, the DSR showed better separation of events and almost no residual reflections as expected, since it can use more traces to construct every sample on the stacked section.

## Conclusions

In the framework of CRS stacking, we propose an approach to reduce the number of parameters to be



Figure 3: PFZ for a ZO diffraction event with pulse length w at a given point  $(m_0, t_0)$ , highlighted in green. Note that the aperture in midpoints for DSR,  $\delta_{Dif}^{(m)}$ , adjusts a much larger region of the diffraction event.



Figure 4: Schematic representation of the proposed apertures in midpoint  $(\delta_{Dif}^{(m)})$  and half-offset  $(\delta_{Dif}^{(h)})$  direction for diffraction enhancement.

estimated in order to obtain a stacked section with reflections or diffractions. The diffraction SSR traveltime, which depends on less parameters than CRS, was investigated to stack reflection events. Stacked sections obtained with SSR with varying apertures were tested and in the case of small apertures in midpoints, the results were slightly better and with lower computational cost to the ones obtained with the conventional CRS with fullparameter (designed for reflections). In both cases, reflections are enhanced and diffractions attenuated.

Diffraction enhancement using SSR and DSR traveltimes were compared with varying apertures. The DSR with large midpoint and offset apertures produced cleaner stacked sections in which diffractions are enhanced and reflections attenuated. In addition, the quantification of small and large apertures was defined using the PFZ for optimal imaging of reflections and diffractions.



Figure 5: Comparison between three stacked traces, CMP's 750, 1000 and 1250, with minimum aperture in midpoint direction and large aperture in offset direction, obtained with the CRS traveltime, estimating A, B and C (solid red line) and with SSR traveltime, estimating A and C (dashed blue line).

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Figure 6: Reflection enhancement: Comparison between CRS (top) and SSR (bottom) stacked sections with minimum apertures in midpoints and large apertures in offset direction. Note that SSR showed less high-frequency noise.



Figure 7: Reflection enhancement: Stacked section obtained with the CRS traveltime using the double of the minimum aperture in midpoints. Note that the reflections were smoothed and resolution compromised.

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Figure 8: Diffraction enhancement: Comparison between SSR (top) and DSR (bottom) stacked sections with the same apertures in midpoint direction. Note that DSR obtained better separation of events and almost no residual reflections.

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